Short Papers

Measured Odd- and Even-Mode Dispersion of Coupled Microstrip Lines

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Abstract—A convenient method is described for the measurement of the odd- and even-mode phase velocities of coupled microstrip lines. Results are presented which show the strong influence on the dispersion characteristics of a closely spaced low-loss shielding cover. The method also enables the loss of the odd and even modes to be determined.

Use of the parallel-coupled-line section in the realization of circuits in microstrip is complicated by the inequality of the odd- and even-mode phase velocities which results from the inhomogeneous structure. Accurate data on the velocities are needed for the proper evaluation of microstrip designs.

In this short paper we describe a convenient technique which has been used to measure the odd- and even-mode phase velocities of coupled microstrip in the frequency range of 4–12 GHz. Measurements have been reported previously by Napoli and Hughes [1]; however, they employ a relatively complex arrangement requiring four connections to the microstrip sample, and the variation of velocity with frequency is not considered. Measured dispersion characteristics have been presented by Gould and Tolboys [2] employing a coupled-line ring resonator. While the ring resonator has the advantage of an absence of end effects, the curvature is known to affect the dispersion [3], [4], and in the case of the coupled-line configuration, the curvature results in the added complication of a difference in the length of the two lines. Our attention has therefore been confined to linear sections.

A linear section of parallel-coupled lines of length L, terminated in open ends will have odd-mode resonances at frequencies f given by

$$L + \Delta_o = nU_o/2f$$

where U_o is the odd-mode phase velocity, and Δ_o represents the increase in effective length due to the odd-mode fringing field at the open ends, and n is an integer. A similar relation, involving corresponding quantities Δ_e,U_e will characterize the even-mode resonances. If two coupled-line resonator systems of lengths L_1,L_2 are fabricated such that two odd-mode (or two even-mode) resonances are obtained with known n_1,n_2 at frequencies f_1,f_2 sufficiently close that dispersive effects can be neglected, then a pair of equations of the form given previously may be used to derive Δ_o,U_o (or Δ_e,U_e) from the measured lengths and frequencies.

The coupled-line resonators were coupled to a 50-Ω input line by a series-gap capacitor, as shown in Fig. 1(a). Resonance of both the odd- and even-mode forms could be determined by observing the reflections at the input line by means of a network analyzer. This simple arrangement does not enable the selective, independent excitation of odd- or even-mode resonance as in the method of [1], but for the parameters encountered in the present work, the error is small. Thus for a Q factor exceeding 100, odd- and even-mode resonances separated by 10 percent in frequency, and a characteristic impedance ratio $Z_{0e}/Z_{0o} = 2$, the error in determining the odd-mode resonance, due to the off-resonance even-mode response, is less than 0.05 percent. It was, however, considered necessary to take into account the asymmetric disturbance of the fringing field due to the presence of the coupling gap, as this could result in coupling between the odd- and even-mode resonances, with an effect depending on the value of n. In order to compensate for this, the resonator strips at the coupling gap are staggered by the difference between the end effect at an open end and the end effect at a coupling gap which had previously been determined [4] for a single strip. The use of data for the single strip situation for the present case is of course an approximation, and, as a check, a representative number of measurements were made with the configurations of Fig. 1(b) and (c). Here an auxiliary section of an effective length of approximately a quarter-wave is coupled to the right-hand end of the structure by a similar gap. The end effects at the two ends of the resonator system are now similar, but the sense of any asymmetry at the right-hand end will be appropriate in Fig. 1(b), (c). There was no significant difference in results obtained from the three configurations, and it was concluded that the arrangement of Fig. 1(a) did satisfactorily prove the symmetry of the resonator system.

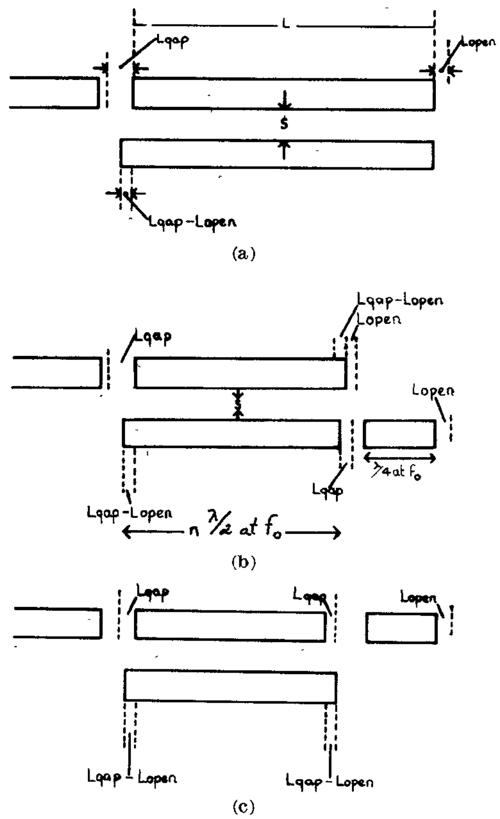

Fig. 1. Test circuits for odd- and even-mode phase velocity measurements.

Circuits were fabricated on 99.5-percent alumina, $\epsilon_r = 9.8$, dimensions 25 × 9.5 × 0.66 mm, using sputtered copper films approximately 3 μm thick. Measurements could be made with the structure open, or within shielding boxes of various heights and sufficiently narrow (10 mm) that rectangular waveguide resonator modes were not excited. Measurements were made with $n = 1$ and 2 near 10 GHz, and the odd- and even-mode velocities and end effects determined. Experience [5] indicated that the variation of end effects over the present frequency range would not be significant, so it was sufficient

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to measure resonator systems with fundamental resonances near 4, 6, and 8 GHz to determine the variation of the velocities over the range 4–12 GHz. Employing the configuration of Fig. 1(a), several measurements can be obtained by etching back the length, in successive stages, of a single basic circuit fabrication.

Results are presented in Fig. 2 for two spacings $s = 0.2$ mm ($\approx$10-dB coupling) and $s = 1.2$ mm ($\approx$20-dB coupling), the strip width, $w = h = 0.660$ mm, being close to that giving 50-Ω impedance for a single strip. There was no significant difference between the results presented for an open structure and those for one with an 8-mm-high shielding cover. The data labeled "closely shielded" were obtained with a shielding cover spaced above the substrate by the substrate height, 0.6 mm. This structure is of interest as in the low-frequency limit it has an effective dielectric constant $(\epsilon_r + 1)/2$ for both modes of propagation, independent of strip width and spacing [1], [6].

Two aspects of the results are of major interest. First, there is relatively little dispersion of the odd mode compared to the even mode, especially for the narrower spacing, so that the inequality of velocity increases with frequency. Secondly, the dispersion is greatly enhanced in the closely shielded structure, particularly for the even mode, indicating a severe limitation of the practical utility of this method of equalizing the phase velocities.

Extrapolating the velocities to zero frequency gives good agreement, within 0.5 percent, with theoretical quasi-static data [7], [8] for the open structure, and acceptable agreement for the closely shielded structure. The greater discrepancy in the latter case, 3 percent, is probably due to inaccuracy in the substrate-to-lid dimension, and the greater extent of the extrapolation.

Comparison of the dispersion characteristics with published results is unfortunately limited by the lack of detailed data for similar

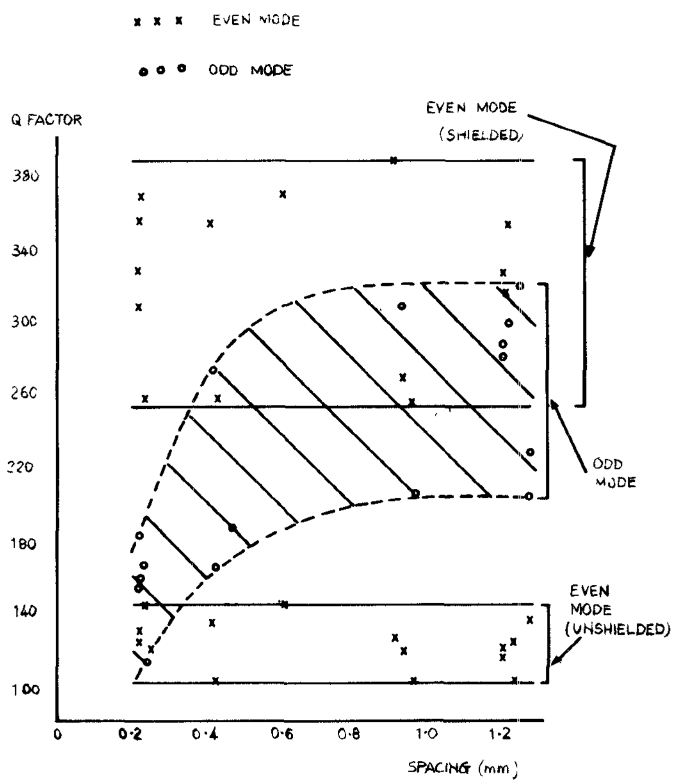

Fig. 3. Measured odd- and even-mode loss.

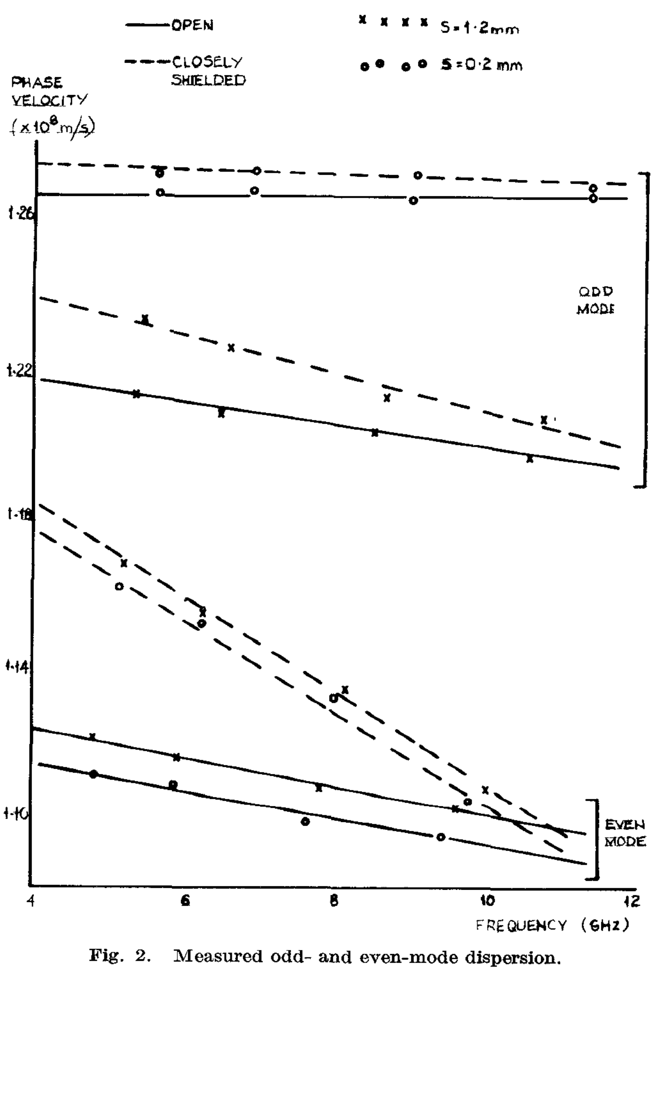

Fig. 2. Measured odd- and even-mode dispersion.

structural parameters. However, the trends indicated in the results of Fig. 2 and outlined previously are in complete agreement with the theoretical results of Krage and Haddad [9] and also with the measured results for an open ring resonator of [2].

The odd- and even-mode loss can also be determined by measurement of the Q factor when determining the resonant frequency. Observing the reflective display, with a Smith chart overlay, the frequency can be set to the points where the normalized resistance and reactance are equal, where the unloaded Q factor can be derived. Results are presented in Fig. 3 for a resonator system with fundamental resonances near 10 GHz and for a range of spacings between 0.2 and 1.2 mm. Measurements were made with the structure open, and with a shielding cover 8 mm high. It will be seen that the even-mode Q factor is not significantly dependent on the spacing, but is appreciably reduced in the absence of the shield due to radiation. The relatively large scatter of experimental points is believed to be due to variations of substrate finish and thin-film quality. The odd mode is associated with much less radiation, and is little affected by the presence or absence of the shield. However, the odd-mode Q falls with decreasing spacing due to the decrease of Z_{0o} and the increasing current concentration in the adjacent strip edges. These results offer confirmation of the theoretical prediction of the different behavior of the odd- and even-mode loss [10].

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A Technique for Determining the Local Oscillator Waveforms in a Microwave Mixer

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Abstract—**A technique is described which enables the large-signal current and voltage waveforms to be determined for a mixer diode. This technique is applicable to any configuration where the impedance seen by the diode at the local oscillator (LO) frequency and its harmonics is known.**

I. INTRODUCTION

The performance of a diode mixer is largely determined by the current and voltage waveforms produced at the diode by the local oscillator (LO). These waveforms depend on the diode itself, and on the impedance of its embedding network at the LO frequency and its harmonics. This short paper describes a method for computing the diode current and voltage waveforms for any mixer in which the impedance seen by the diode at the LO frequency and its harmonics is known.

In the past there have been various approaches to this problem. Torrey and Whitmer [1] and others have assumed a sinusoidal driving voltage at the diode, all the harmonics of the LO being assumed to be short-circuited. Fleri and Cohen [2] used both digital and analog computers to solve the nonlinear problem, assuming simple lumped-element embedding networks.

Egami [3] and Gwarek [4] have used a harmonic balance approach in the frequency domain. However, convergence has been found difficult to achieve for some circuits when many harmonics are considered, and especially at large LO drive levels; and the initial guess has a strong effect on the rate of convergence.

A recent approach by Gwarek [4] uses a time-domain analysis to determine the diode waveforms in an embedding network consisting of a simple lumped-element network in series with a string of voltage sources, one at each harmonic of the LO. The voltage sources are input-voltage dependent so that the embedding network is able to simulate any complex network as it appears at the LO frequency and its harmonics. It is reported that this method is convergent, and more economical of computer time and memory than the harmonic balance technique.

In the approach described here, the circuit of Fig. 1(a) is modified

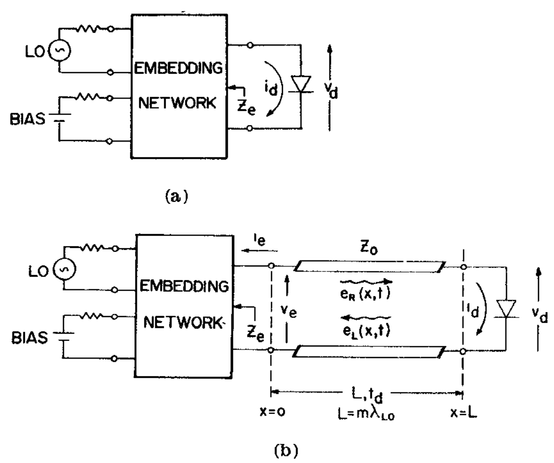

Fig. 1. (a) The mixer circuit for which v_d and i_d are to be determined. (b) The modified circuit which has the same steady-state v_d and i_d provided L is an integral number of wavelengths at the LO frequency. The right- and left-propagating waves on the transmission line are denoted by e_R and e_L.

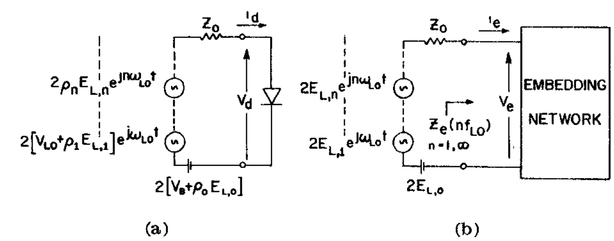

Fig. 2. The two circuits whose steady-state solutions are alternately computed to determine the steady-state solution of Fig. 1(b). The simple nonlinear circuit (a) is solved in the time domain, the linear circuit (b) in the frequency domain. Source amplitudes are given by (9)–(11).

by the insertion of a transmission line, Fig. 1(b), which, by virtue of its electrical length at the LO frequency and its harmonics, has no effect on the steady-state solution of the problem. It will be shown that this enables the problem to be solved iteratively by alternately solving the simpler circuit problems shown in Fig. 2(a) and (b).

Hypothesis: The steady-state i_d and v_d waveforms for the two circuits of Fig. 1 are the same. Certainly the solution for Fig. 1(b) is a valid solution for Fig. 1(a), but there exists the possibility of more than one steady-state solution for Fig. 1(a), the one which is finally reached being determined by the particular path taken to reach the steady state.[1] An hypothesis equivalent to this one is implicit in any method of solution in which the calculation of the actual turn-on transient is bypassed.

II. METHOD

Consider the circuit of Fig. 1(b) with the diode initially disconnected. When the diode is connected transient reflections occur alternately at the two ends of the transmission line until eventually the steady-state condition is approached. In the steady state, waves of constant amplitude, containing many LO harmonics generated by the diode, propagate in each direction. The approach taken here is to let the transmission line become so long that in the periods between transient reflections a steady-state condition is reached

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[1] As an example of this, it has been observed that for some mixers, as the LO power is increased from zero, parametric oscillation will occur. At higher power levels the oscillation ceases, and it cannot be made to reappear unless the LO power is first reduced below some threshold value and then increased again.